

Forward Kinematics

$${}^0T_4 = [{}^0T_1][{}^1T_2][{}^2T_3][{}^3T_4] \quad H_{ee} = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

H_{ij} - function of θ_i $f_i(\theta_i)$ $K=6 \rightarrow 0$

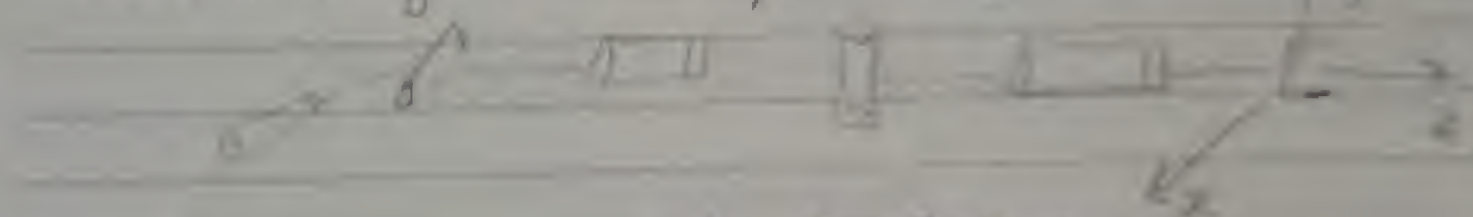
Inverse Kinematics multiple solutions $\theta_{ik} = g_n(P_{des})$
given desired pose

Pose $\begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$ for θ_k

given H_{ee} desired
forward kinematics

model from solving 12 nonlinear eqns

H_{ij} - constant values



$H_{ee} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -12 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $P_{des} = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$
 $P_{12} E_{12} = 0$
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12 nonlinear eqns

Calculated DOF \rightarrow DOF

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1. Closed form

analytical method

$q = f(t)$

1. given data

kinematic chain

2. closed form solution of the kinematic chain

3. closed form solution of the kinematic chain

Analytical method
analytical

Geometric

also called

- Simple Kinematic Chain
- Robo Geometry

2. Numerical method

iteration method is given by Forward Kinematics

3. Artificial intelligence

Pose \rightarrow NN \rightarrow f

Artificial Intelligence

Topic: _____

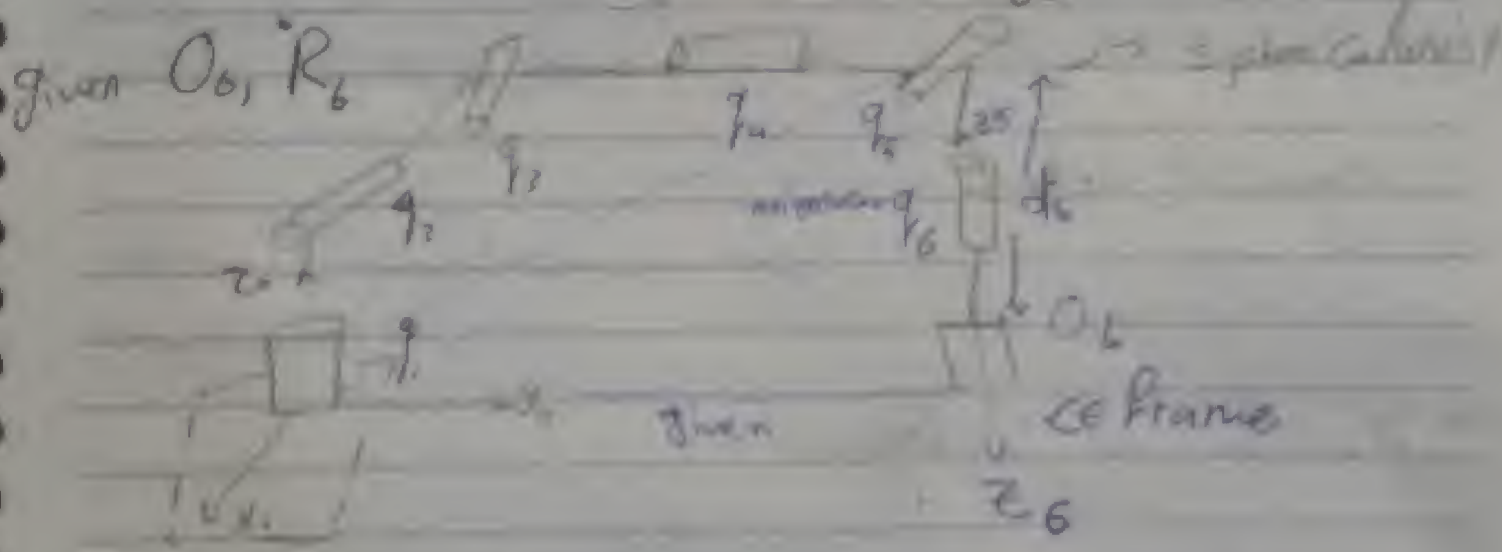
Kinematic Decoupling: Geometric approach

6 DOF manipulator
spherical wrist

Kinematic
Robot

wrist center = $f(q_1, q_2, q_3)$

orientation = $f(q_4, q_5, q_6)$
spherical wrist angles



$$\dot{Q}_0 = \dot{Q}_6 + R_6 d_4 \begin{bmatrix} \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix}$$

$$\dot{Q}_6 = \dot{Q}_0 - d_4^T R_6^T \begin{bmatrix} \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix} \quad \text{--- (1)}$$

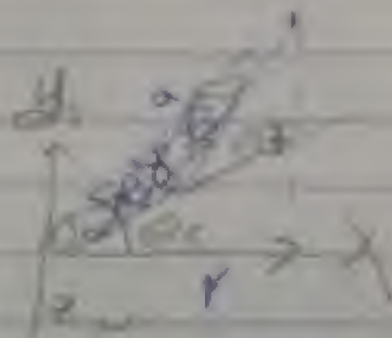
O_0, q_1, q_2, q_3

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θ_1 → هي الزاوية بين قاطع الـ manipulator على المستوى الأفقي

θ_2 →



$$\tan \theta_2 = \frac{y_C}{x_C}$$

$$\theta_2 = \tan^{-1} \left(\frac{y_C}{x_C} \right) \rightarrow (2)$$

$$= \text{atan2}(y_C, x_C) \rightarrow \text{matlab}$$

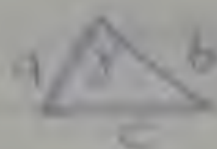
نلاحظ أن الرتبة ونسبة الإشارة بالتغير الأتري

$$r = \sqrt{x_c^2 + y_c^2}$$

$$a^2 = r^2 + (ze - d)^2$$

$$a^2 = x_c^2 + y_c^2 + (ze - d)^2$$

$$\cos \delta = \frac{a^2 + b^2 - c^2}{2ab}$$



$$c^2 = a^2 + b^2 - 2ab \cos \delta$$

$$\cos \delta = \frac{r^2 + r^2 - c^2}{2r^2}$$

cosines law
for all sides and angles

$$\theta_3 = 180 - \delta$$



$$\cos \theta_3 = \frac{r^2 + r^2 + x_c^2 + y_c^2 + (ze - d)^2}{2r^2}$$

$$b = r \cos \theta_3$$

$$B = \cos^{-1} \left(\frac{r^2 + r^2 + x_c^2 + y_c^2 + (ze - d)^2}{2r^2} \right)$$

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$$\theta_2 = \tan^{-1} \frac{z_2 - u_2}{\sqrt{x_2^2 + y_2^2}} \quad R$$

∞

$${}^0R_1 = {}^0R_3 {}^3R_6$$

$${}^3R_6 = ({}^3R_3)^{-1} {}^3R_6$$

$\hookrightarrow \theta_4, \theta_5, \theta_6$
 using Euler Parameterization.

substitute by $\theta_4, \theta_5, \theta_6$